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Mathematical simulation of joint work of a vibration isolator group with quasi-zero stiffness

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The relevance of the work is due to the need to develop new means of highly efficient vibration protection. One such means is a vibration isolator with quasi-zero stiffness. They are quite sensitive elements, so the problem of designing and operating vibration isolators with quasi-zero stiffness is relevant. Nowadays, comprehensive studies on their work within a group have not yet conducted.
Purpose of the work is to study the sensitivity of vibration isolators with quasi-zero stiffness to the errors of geometric parameters while their manufacture.
Methodology of research. This work is a continuation of the experimental studies of plate-type universal vibration isolators with quasi-zero stiffness. For the research, an analytical study and a computer-based multiple experimental procedure with random input data were used.
Results. Analytical studies show that vibration isolators with quasi-zero stiffness are very sensitive objects. Basic properties, such as workload and stiffness under workload, largely depend on key parameters. Vibration isolators of plate type have a very strong dependence of the workload on the external and internal radii, the height of the cone and the wall thickness. The dynamics of a group of vibration isolators was analysed. Due to the deviation of different parameters and the nonlinearity of the power characteristics, the behaviour of the group does not coincide with the average behaviour of one vibration isolator. It has been found that for a group of isolators there is a slight increase in workload. Moreover, deviations in parameters lead to a decrease in stiffness.
Conclusions. The high sensitivity of installed vibration isolators with quasi-zero stiffness proves that they require careful attention and high precision in manufacturing. The resulting deviations of the behaviour of a group of vibration isolators from the behaviour of a single vibration isolator indicate the need to enter appropriate corrections when designing them, otherwise this may lead to loss of stability and instability of the equipment, which should be avoided.

Keywords: vibration, vibration isolator, quasi-zero stiffness, mathematical simulation, nonlinear oscillations, sensitivity.

Introduction

A significant part of modern equipment is characterised by high power, and its work is accompanied by high noise and vibration. These factors negatively affect equipment and personnel.

The high level of vibratory background noise accelerates the wear of equipment, bearing assembly and bases. In addition, noise creates uncomfortable working conditions and can cause chronic diseases [1-3]. Laboratory and high-precision equipment also have the problem of vibration protection. The need for protection from vibration arises in everyday life, for example, in washing machines [4].

Various methods can be used for noise and vibration protection: an increase in the mass of equipment, vibration isolation, and the use of dynamic dampers. From a practical point of view, the most convenient way is vibration isolation. Traditional spring and rubber vibration isolators are well suited only for standard conditions, but their use is complicated under special conditions, in particular, if it is necessary to obtain a high coefficient of vibration isolation with small dimensions.

For high-quality vibration isolation, it is necessary to have a low natural frequency of the entire system. But traditional vibration isolators, in fact, are an elastic linear mechanical system with large dimensions. This problem can be solved with the help of vibration isolators with a low natural frequency.

Vibration isolators with quasi-zero stiffness are elastic nonlinear mechanical systems with a section with a small (or low) stiffness value. Also, these systems are sometimes called “systems with low stiffness” [5]. In foreign literature one can come across the term “systems with high-static-low-dynamic-stiffness” [5, 6]. The principal view of the power characteristic is presented in Fig. 1.

Vibration isolators with quasi-zero stiffness represent a very promising development in mechanical engineering. They can prove well in various areas: industrial machines and equipment, vibration control of heavy vehicles, workstations, manual machines, marine engines, close control and aerospace equipment, etc. Vibration isolators with quasi-zero stiffness provide both high static load and low dynamic stiffness. The low stiffness of the system with a significant static load reduces the natural frequency of oscillation to less than 1 Hz and allows you to get rid of a wide range of vibration with high efficiency.

P. Alabuzhev studied vibration isolators with quasi-zero stiffness. He analysed various types of passive systems with quasi-zero stiffness [5]. A. Carrella is also known for analysing quasi-zero stiffness by means of two inclined springs [6] or two compressed rods [7–9]. The observation of the “scissors” system with a spring for obtaining quasi-zero stiffness was done by X. Sun et al.

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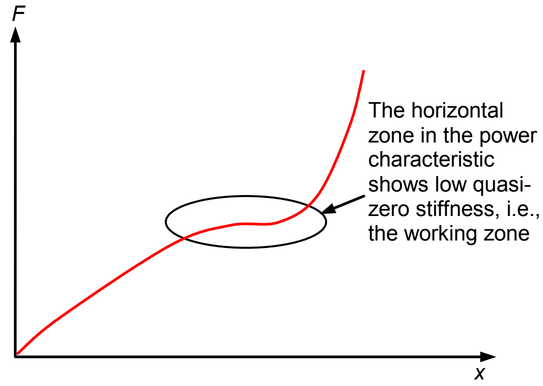

Figure 1. Power characteristic of a system with quasi-zero stiffness.
Рисунок 1. Силовая характеристика системы с квазинулевой жесткостью.

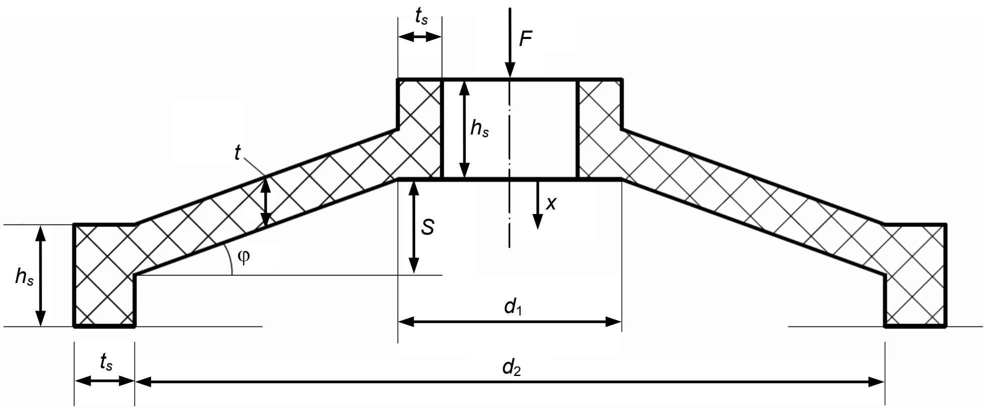

Figure 2. Plate-type vibration isolator with quasi-zero stiffness.
Рисунок 2. Виброизолятор с квазинулевой жесткостью тарельчатого типа.

[10]. These systems can also be obtained using pneumatic active elements, as described in [11]. Pneumatics are also used by M. Holtz and J. Van Niekerk for seat suspension [12]. The application of the quasi-zero stiffness effect for special isolators of cables is studied by Y. K. Ponomarev [13, 14]. Another type of special cable is studied by P. Tapia-Gonzales and others in [15]. In addition, systems with quasi-zero stiffness of the passive type are proposed by T. Le and K. Ahn [16] and I. Matseevsky [17].

There is also a new prototype vibration isolator with quasi-zero stiffness, based on folding paper cylinders with torsional buckling [18]. Experimental research shows that the elimination of oscillations occurs at frequencies greater than 6 Hz. It is also known that it is possible to obtain quasi-zero stiffness with cammed spring mechanisms [19].

Systems with quasi-zero stiffness can also be obtained by active or semi-active methods. A suspension system with semi-active devices was proposed by J. Choi and others [20, 21]. An electromagnetic actuator with linear characteristic is used by Kh. Khan and others [22]. The electric servomotor with a ball-and-screw unit is also actively used as a power suspension drive by M. Kawana and T. Shimogo [23]. Electromagnetic systems are presented by V. Robertson and others [24]. Unfortunately, active systems with quasi-zero stiffness are usually quite expensive. Systems with quasi-zero stiffness, controlled by a rotary drive, are presented by D. Ning et al. [25]. There is also a vibration isolator developed by Y. Zheng and others [26]. It is implemented by connecting a magnetic spring with negative stiffness parallel to the springs of the membranes in order to compensate for its positive stiffness. An isolator consists of two coaxial magnetic rings that have axial magnetisation, and their directions of magnetisation are the same. Quasi-zero stiffness can also be obtained using the suspension system of an off-road vehicle using intelligent active power control [27].

Quasi-zero stiffness can also be used to create an elastic suspension for equipment shafts [28]. In this case, it is possible to minimise the transmission of dynamic effects from the rotor to the pump or compressor casing.

Often, systems with quasi-zero stiffness are characterised by design complexity. Tests by authors and other scientists show that the practical implementation of systems with quasi-zero stiffness is rather complicated. Any significant parameter deviation may impair the operation of the vibration isolator. It is also possible that serially manufactured vibration isolators installed in one equipment may differ, and, accordingly, their behaviour may slightly deviate from the predicted one.

As follows from the experience of the authors and other scientists, vibration isolators with quasi-zero stiffness are, on the whole, quite sensitive to changes in geometric parameters. This leads to the need for careful adjustment. The use of such a vibration isolator requires studying the possibility of deviating the parameters and analysing their influence on the effectiveness of vibration isolation. Moreover, usually prototypes of vibration isolators with quasi-zero stiffness are developed separately. But it is obvious that at least four vibration isolators are required for machines or equipment, to ensure better stability as well. It is important to know the difference in behaviour between one vibration isolator and a group of vibration isolators.

Thus, the purpose of this study is to analyse the sensitivity of the characteristics of a vibration isolator with quasi-zero stiffness when changing geometric parameters, as well as analysing the behaviour of a group of vibration isolators with several different parameters.

Mathematical model of plate-type vibration isolator with quasi-zero stiffness

A tray-type spring of a disc without any changes in the design is difficult to apply, because it has a quasi-zero stiffness when it is flat (the height of the cone h is zero), i.e. fully incorporated into the base. On the other hand, the tray-type spring should have room for deformation; therefore, something like support should be used. This design of an isolator is shown in Fig. 2. To distinguish these two designs, an isolator in Fig. 2 is called "dome-shaped."

For further description of the vibration isolator with quasi-zero stiffness, the following parameters are used (Table 1). The detailed derivation of the formulas in this paper is not shown.

The relationship between the load F on the vibration isolator and its compression x is determined by the formula [29]:

$$F(x)=\pi tE\left(\frac{t^2}{(r_2-r_1)\left(r_1+r_2(2k_v-3)\right)+2r_2^2(1-k_v)\ln(r_2/r_1)}\cdot\frac{x}{3}+\frac{1}{4\cdot\ln(r_2/r_1)}\cdot\frac{8S^2x-12Sx^2+4x^3}{(r_2-r_1)^2}\right). \tag{1}$$

The optional coefficients k_s and k_v are described as

$$k_s=\frac{t^3d_2}{8t_sh_s^3}; \tag{2}$$

$$k_v=\frac{\ln(r_2/r_1)-\left(1-(r_1/r_2)\right)}{k_s+\ln(r_2/r_1)}. \tag{3}$$

The height of the cone wall

$$S=\frac{1}{\sqrt{3}}t(r_2-r_1)\sqrt{\frac{\ln(r_2/r_1)}{(r_2-r_1)\left(r_1+r_2(2k_v-3)\right)+2r_2^2(1-k_v)\ln(r_2/r_1)}}. \tag{4}$$

Operating load

$$F=\frac{\pi/(3\sqrt{3})t^4E(r_2-r_1)\sqrt{\ln(r_2/r_1)}}{\left((r_2-r_1)\left(r_1+r_2(2k_v-3)\right)+2r_2^2(1-k_v)\ln(r_2/r_1)\right)^{1,5}}. \tag{5}$$

Maximum stress in the wall of the vibration isolator

$$\sigma=\frac{F}{(\pi/3)t^2}\left(\frac{r_2(1-k_v)}{r_1}-1\right). \tag{6}$$

For further analysis, the stiffness of the vibration isolator should be taken into account:

$$k=\frac{dF}{dx}.$$

Table 1. Description of parameters of a plate-type isolator with stiffness.
Таблица 1. Описание параметров изолятора с квазинулевой жесткостью тарельчатого типа.

Criterion	Physical meaning
Compression of vibration isolator x, m	Variable
Capacity of vibration isolator $F(x)$, H	Elastic force of vibration isolator
Operating load of vibration isolator F, H	Capacity of vibration isolator with minimum hardness
Thickness of the cone t, m	Geometric parameter of vibration isolator
Inner radius of vibration isolator r_1, m	Geometric parameter of vibration isolator
Outer radius of vibration isolator r_2, m	Geometric parameter of vibration isolator
Young's modulus E, Pa	Describes the properties of an isolator material
Optional coefficient k_s	Characterises the shape of the outer wall for a thick or rigid wall. The value k_s tends to zero.
Optional coefficient k_v	Shows the effect of the outer wall on hardness. For a thin outer wall, the value k_v tends to zero.
Maximum height of the inner cone S, m	Geometric parameter of vibration isolator
Pressure wall thickness t_s, m	Geometric parameter of vibration isolator
Height of the sustaining wall h_s, m	Geometric parameter of vibration isolator
Stress σ, PA	Maximum stress in vibration isolator in static condition
Hardness k, N/m	Hardness of the vibration isolator in the direction of compression

After changes:

$$k = \pi E t\left[\frac{t^2/3}{(r_2-r_1)(r_1+r_2(2k_v-3))+2r_2^2(1-k_v)\ln(r_2/r_1)}+\frac{1}{\ln(r_2/r_1)}\cdot\frac{2S^2-6Sx+3x^2}{(r_2-r_1)^2}\right]. \quad (7)$$

Stiffness of vibration isolator at the operating point

$$k = \pi E t\left[\frac{t^2/3}{(r_2-r_1)(r_1+r_2(2k_v-3))+2r_2^2(1-k_v)\ln(r_2/r_1)}-\frac{1}{\ln(r_2/r_1)}\cdot\frac{S^2}{(r_2-r_1)^2}\right]. \quad (8)$$

These formulas are used for further analysis.

The sensitivity of the vibration isolator with quasi-zero hardness to changes in geometric parameters

The design of vibration isolators with quasi-zero hardness and the study of the dynamics of a group of vibration isolators require information about sensitivity to the deviation of their parameters.

Let us consider a situation in which one of the parameters of the isolator differs sharply from its calculated value. We determine the change in workload and stiffness when changing geometric parameters. Based on the analysis of formulas (1)–(8), the sensitivity of dimensionless parameters is determined, which is summarised in Table 2. It shows the deviations of the workload with respect to various geometric parameters of the vibration isolator (sensitivity of the workload). The specified parameters are: wall thickness, height of the cone, inner radius of the isolator, outer radius of the isolator, thickness of the outer wall of the vibration insulator and height of the outer wall of the vibration insulator. Calculations are made in dimensionless form. The following dimensionless parameters were used:

– dimensionless workload $\bar{F} = \frac{F}{Er_2^2}$;

– dimensionless wall thickness $\bar{t} = \frac{t}{r_2}$;

– dimensionless inner radius $\bar{r} = \frac{r_1}{r_2}$;

– dimensionless height of the cone $\bar{S} = \frac{S}{r_2}$;

– dimensionless thickness of the outer wall $\bar{t}_s = \frac{t_s}{r_2}$;

– dimensionless height of the outer wall $\bar{h}_s = \frac{h_s}{r_2}$;

– dimensionless stiffness $\bar{k} = \frac{k}{F/S}$.

Table 2. Sensitivity of the workload.
Таблица 2. Чувствительность рабочей нагрузки.

Dimensionless deviation	Deviation of dimensionless value					
	Wall thickness t	Cone height S	Inner radius r_1	Thickness of the outer wall t_s	Height of the outer wall h_s	Outer radius r_2
–0.10	–0.1840	–0.1000	–0.3591	–0.0372	–0.1068	1.5361
–0.09	–0.1662	–0.0900	–0.3316	–0.0333	–0.0963	1.2814
–0.08	–0.1484	–0.0800	–0.3026	–0.0295	–0.0857	1.0591
–0.07	–0.1304	–0.0700	–0.720	–0.0258	–0.0751	0.8643
–0.06	–0.1122	–0.0600	–0.2395	–0.0220	–0.0644	0.6928
–0.05	–0.0939	–0.0500	–0.2052	–0.0183	–0.0538	0.5414
–0.04	–0.0755	–0.0400	–0.1689	–0.0146	–0.0431	0.4071
–0.03	–0.0569	–0.0300	–0.1304	–0.0109	–0.0323	0.2876
–0.02	–0.0381	–0.0200	–0.0895	–0.0072	–0.0216	0.1810
0.01	–0.0191	–0.0100	–0.0461	–0.0036	–0.0108	0.0856
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0193	0.0100	0.0491	0.0036	0.0108	–0.0771
0.02	0.0388	0.0200	0.1013	0.0071	0.0216	–0.1466
0.03	0.0584	0.0300	0.1571	0.0107	0.0324	–0.2095
0.04	0.0783	0.0400	0.2166	0.0142	0.0432	–0.2665
0.05	0.0983	0.0500	0.2801	0.0177	0.0540	–0.3183
0.06	0.1186	0.0600	0.3481	0.0212	0.0647	–0.3654
0.07	0.1390	0.0700	0.4210	0.0246	0.0755	–0.4085
0.08	0.1597	0.0800	0.4991	0.0280	0.0862	–0.4478
0.09	0.1805	0.0900	0.5830	0.0315	0.0969	–0.4838
0.10	0.2016	0.1000	0.6732	0.0348	0.1076	–0.5168
Sensitivity	1.9210	1.0000	4.7600	0.3600	1.0790	–8.1350

The sensitivity of hardness at the operating point (i.e. the minimum hardness of the system) is presented in Table 3 (similarly, the change in hardness when changing one of the geometric parameters of the vibration isolator). The hardness sensitivity is calculated relative to the static stiffness, i.e. with respect to the value equal to the value of the workload divided by the working compression of the vibration isolator (i.e. F/S).

Table 3. Stiffness sensitivity at the working point.
Таблица 3. Чувствительность жесткости в рабочей точке.

Dimensionless deviation	Deviation of dimensionless value					
	Wall thickness t	Cone height S	Inner radius r_1	Thickness of the outer wall t_s	Height of the outer wall h_s	Outer radius r_2
–0.10	–0.1029	0.0266	0.0077	–0.0054	–0.0167	–0.0055
–0.09	–0.0914	0.0241	0.0070	–0.0048	–0.0149	–0.0047
–0.08	–0.0803	0.0215	0.0063	–0.0043	–0.0131	–0.0039
–0.07	–0.0694	0.0189	0.0055	–0.0037	–0.0114	–0.0033
–0.06	–0.0588	0.0163	0.0048	–0.0031	–0.0096	–0.0027
–0.05	–0.0485	0.0137	0.0040	–0.0026	–0.0080	–0.0021
–0.04	–0.0384	0.0110	0.0032	–0.0021	–0.0063	–0.0016
–0.03	–0.0285	0.0083	0.0024	–0.0015	–0.0047	–0.0012
–0.02	–0.0188	0.0055	0.0016	–0.0010	–0.0031	–0.0007
–0.01	–0.0093	0.0028	0.0008	–0.0005	–0.0015	–0.0004
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0091	–0.0028	–0.0008	0.0005	0.0015	0.0003
0.02	0.0181	–0.0057	–0.0017	0.0010	0.0030	0.0006
0.03	0.0269	–0.0085	–0.0026	0.0015	0.0044	0.0009
0.04	0.0355	–0.0114	–0.0035	0.0020	0.0058	0.0011
0.05	0.0440	–0.0143	–0.0044	0.0024	0.0072	0.0013
0.06	0.0524	–0.0173	–0.0053	0.0029	0.0085	0.0015
0.07	0.0606	–0.0203	–0.0063	0.0034	0.0098	0.0017
0.08	0.0687	–0.0233	–0.0072	0.0038	0.0111	0.0019
0.09	0.0767	–0.0263	–0.0082	0.0043	0.0124	0.0020
0.10	0.0845	–0.0294	–0.0092	0.0047	0.0136	0.0021
Sensitivity	0.9210	–0.2800	–0.0840	0.0500	0.1510	0.0340

The sensitivity of the workload and stiffness at the operating point is shown in Fig. 3, 4.

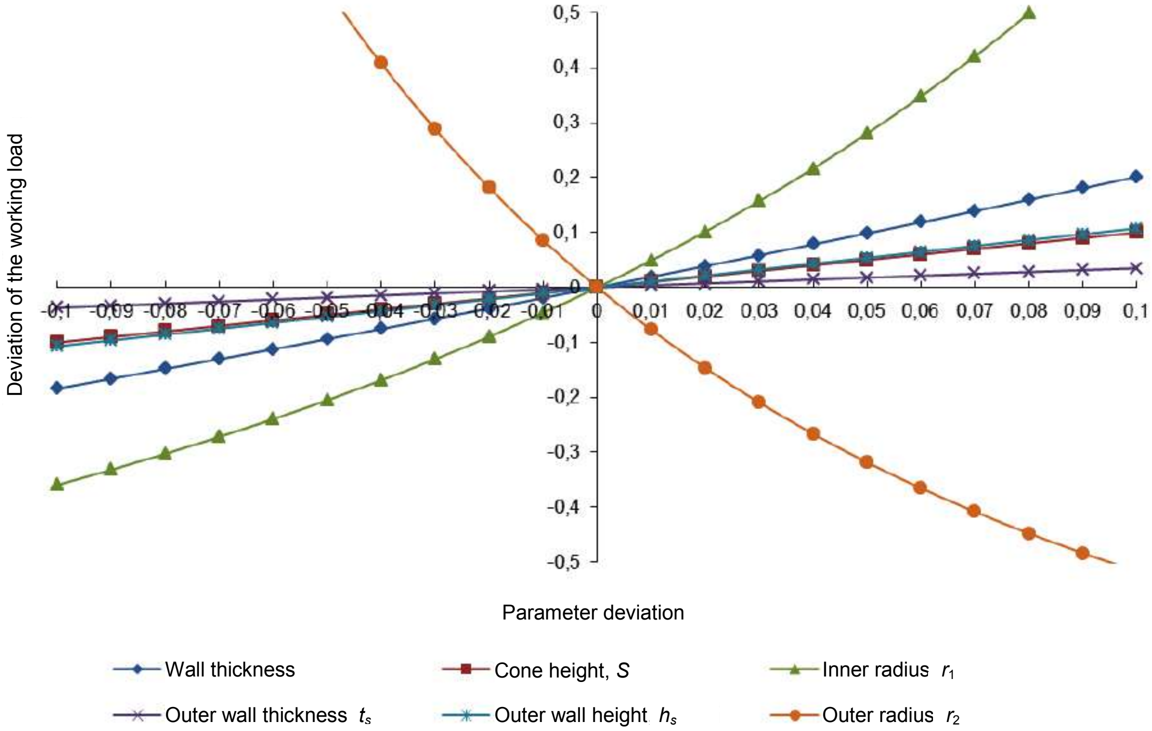

Figure 3. Sensitivity of the workload.
Рисунок 3. Чувствительность рабочей нагрузки.

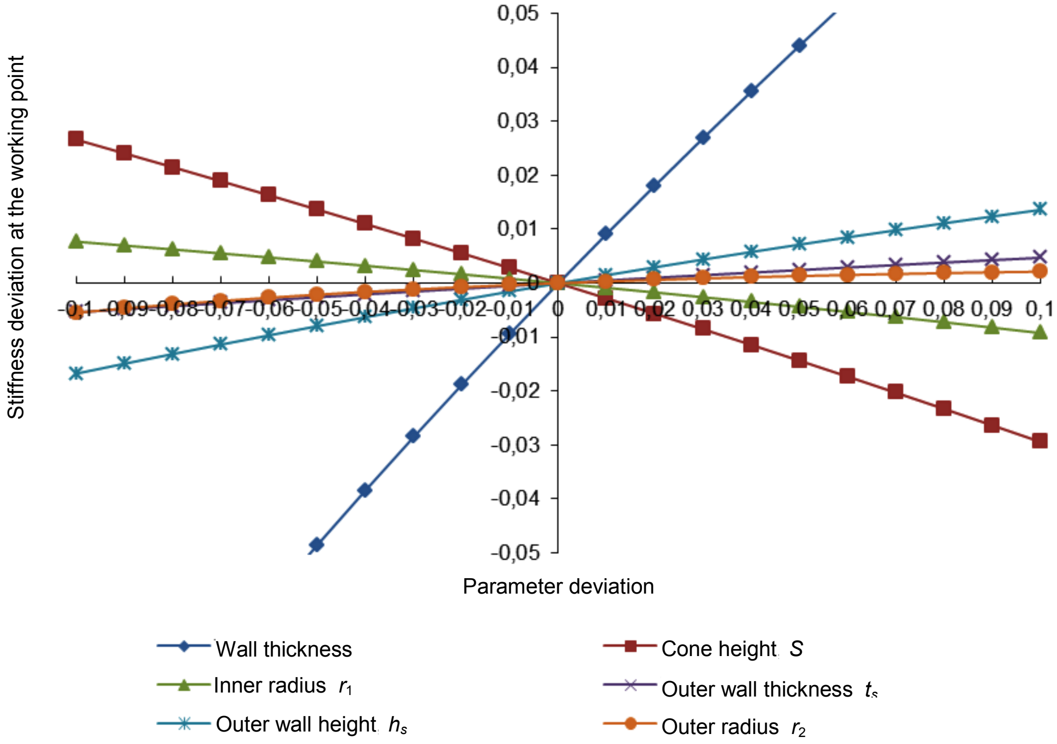

Figure 4. Stiffness sensitivity at the working point.
Рисунок 4. Чувствительность жесткости в рабочей точке.

Table 4. Deviations for a group of vibration isolators.
Таблица 4. Отклонения для группы виброизоляторов.

Standard deviation of dimensionless parameters	Deviation of average dimensionless workload	Standard deviation of dimensionless workload	Deviation of the average dimensionless stiffness under workload	Standard deviation of dimensionless stiffness under workload
0.000	0.00000	0.0000	0.0000	0.0000
0.005	0.00057	0.0267	0.0000	0.0127
0.010	0.00114	0.0534	–0.0006	0.0258
0.015	0.00256	0.0802	–0.0006	0.0383
0.020	0.00603	0.1099	–0.0014	0.0513
0.025	0.00962	0.1365	–0.0028	0.0643
0.030	0.01322	0.1650	–0.0030	0.0789
0.035	0.01935	0.1925	–0.0067	0.0934
0.040	0.02491	0.2265	–0.0072	0.1090
0.045	0.03079	0.2553	–0.0064	0.1217
0.050	0.04026	0.2884	–0.0077	0.1390
0.055	0.04811	0.3214	–0.0123	0.1571
0.060	0.06269	0.3628	–0.0138	0.1745
0.065	0.07894	0.4122	–0.0196	0.1927
0.070	0.08751	0.4538	–0.0200	0.2200
0.075	0.09356	0.4920	–0.0229	0.2367
0.080	0.10555	0.5215	–0.0265	0.2636
0.085	0.12513	0.6143	–0.0339	0.2971
0.090	0.13879	0.6486	–0.0334	0.3126
0.095	0.16731	0.7328	–0.0479	0.3507
0.100	0.18932	0.7776	–0.0462	0.4064

As can be seen from Table 2 and Fig. 3, the value of the workload is very sensitive to the internal and external radii of the vibration isolator. It is also sensitive to wall thickness and cone height. Consequently, these parameters should be given special attention in the production of vibration isolators.

In accordance with Table 3 and Fig. 4, one can see that stiffness at the working point strongly depends on the wall thickness and is inversely related to it. It should be noted that if the wall thickness is less than the calculated one, then the stiffness can become negative. This means that a "snap buckling" will occur, i.e. stability loss, which is unacceptable for the vibration isolator. Therefore, the wall thickness must be carefully controlled.

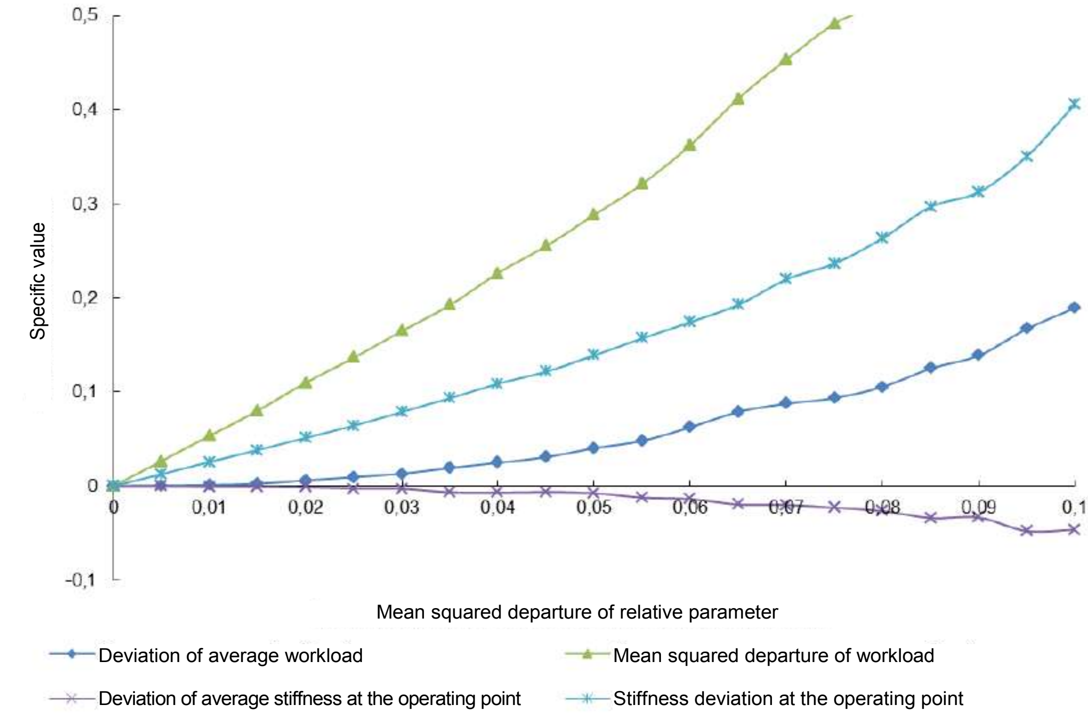

Figure 5. Stiffness sensitivity at the working point.
Рисунок 5. Чувствительность жесткости в рабочей точке.

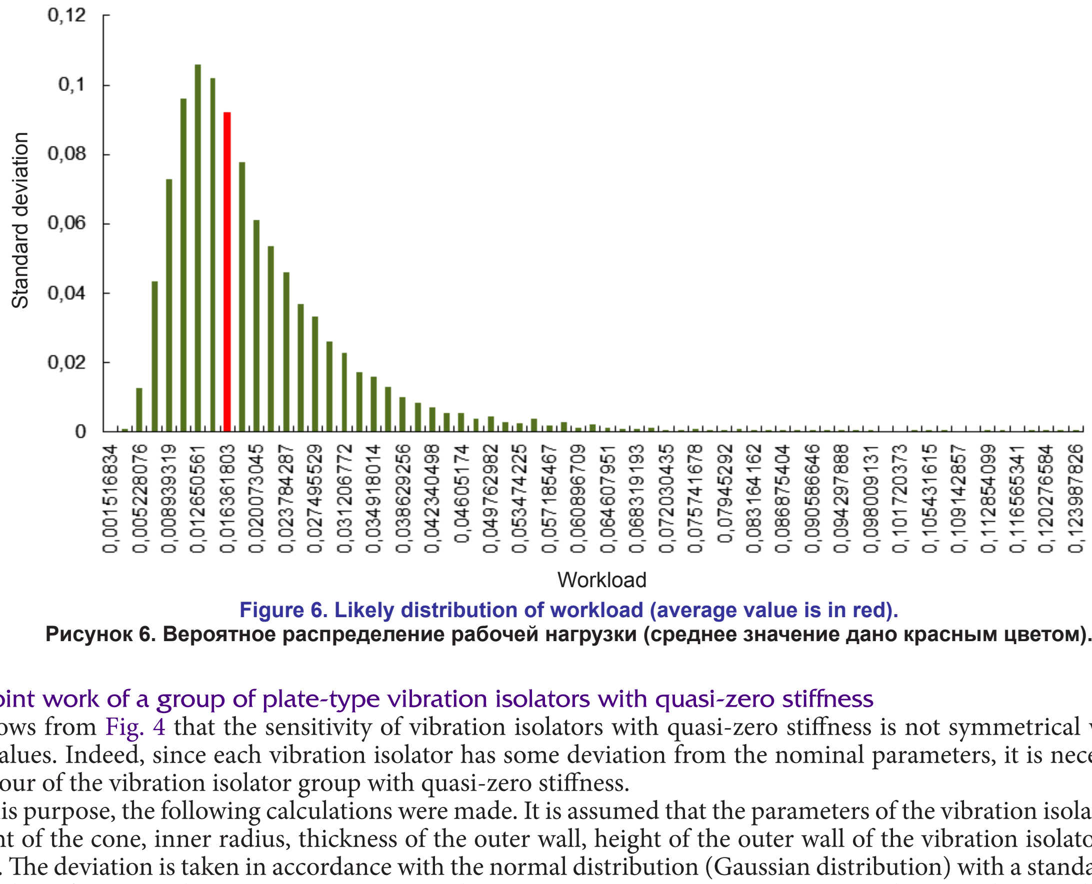

Figure 6. Likely distribution of workload (average value is in red).
Рисунок 6. Вероятное распределение рабочей нагрузки (среднее значение дано красным цветом).

The joint work of a group of plate-type vibration isolators with quasi-zero stiffness

It follows from Fig. 4 that the sensitivity of vibration isolators with quasi-zero stiffness is not symmetrical with respect to nominal values. Indeed, since each vibration isolator has some deviation from the nominal parameters, it is necessary to check the behaviour of the vibration isolator group with quasi-zero stiffness.

For this purpose, the following calculations were made. It is assumed that the parameters of the vibration isolator (wall thickness, height of the cone, inner radius, thickness of the outer wall, height of the outer wall of the vibration isolator) may deviate somewhat. The deviation is taken in accordance with the normal distribution (Gaussian distribution) with a standard deviation σ.

A number of numerical experiments were carried out. During one experiment, 10 000 isolators with a standard deviation σ were calculated. The results are presented in Table 4.

The results from Table 4 are graphically illustrated in Fig. 5.

The analysis of Fig. 6 shows that the average workload (or the total workload of an isolator group) increases with an increase in the standard deviation of the parameters. Therefore, it is necessary to develop isolators for a lower workload. Moreover, as the number of vibration isolators increases, the spread of geometric parameters increases, and the standard deviation increases. Therefore, the dynamics of the vibration isolator group do not coincide with the behaviour of a single vibration isolator.

As for stiffness, its value decreases. Stiffness less than zero is unacceptable for isolators, as it is possible to observe a loss of stability. It leads to unacceptable system behaviour, increased vibration, and reduced durability.

To avoid the loss of stability of all systems, the stiffness of all vibration isolators should be slightly increased. For example, for the standard deviation of dimensionless parameters $\sigma = 0.1$, stiffness decreases by 4.6% (Table 4). Thus, to restore the stiffness value, the height of the cone should be reduced by 2.6% (according to Table 3).

Conclusion

Analytical studies prove the fact that vibration isolators with quasi-zero stiffness are very sensitive objects. Basic properties, such as workload and stiffness under workload, largely depend on key parameters. In plate-type isolators, there is a very strong dependence of the working load on the external and internal radii, the height of the cone and the wall thickness. This proves that vibration isolators with quasi-zero stiffness require careful attention and high precision in manufacturing.

The dynamics of a group of vibration isolators was analysed. Due to the deviation of different parameters and the nonlinearity of the power characteristics, the behaviour of the group does not coincide with the average behaviour of one vibration isolator. It turns out that for a group of isolators there is a slight increase in workload. But another problem is more important. Deviations of parameters lead to a decrease in stiffness, which can lead to loss of stability and instability of equipment Therefore, such deviations should be avoided. To this end, the reduction in stiffness due to a decrease in the height of the cone should be compensated.

Thus, the effectiveness of vibration isolators with quasi-zero stiffness can be increased through careful analysis and design.

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Математическое моделирование совместной работы группы виброизоляторов с квазинулевой жесткостью

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Актуальность работы обусловлена необходимостью разработки новых средств высокоэффективной вибрационной защиты. Одними из таких средств являются виброизоляторы с квазинулевой жесткостью. Они являются достаточно чувствительными элементами, поэтому проблема проектирования и эксплуатации виброизоляторов с квазинулевой жесткостью является актуальной. На данный момент не проведены еще комплексные исследования по их работе в группе.
Целью работы является исследование чувствительности виброизоляторов с квазинулевой жесткостью к погрешностям геометрических параметров при их изготовлении.
Методология исследования. Данная работа является продолжением экспериментальных и опытных исследований универсальных виброизоляторов с квазинулевой жесткостью тарельчатого типа. Для исследования применялось аналитическое исследование и компьютерный многократный эксперимент со случайными входными данными.
Результаты. Аналитические исследования показывают, что виброизоляторы с квазинулевой жесткостью являются очень чувствительными объектами. Основные свойства, такие как рабочая нагрузка и жесткость при рабочей нагрузке, в значительной степени зависят от ключевых параметров. У виброизоляторов тарельчатого типа наблюдается очень сильная зависимость рабочей нагрузки от внешних и внутренних радиусов, высоты конуса и толщины стенки. Была проанализирована динамика группы виброизоляторов. Из-за отклонения разных параметров и нелинейности силовых характеристик поведение группы не совпадает со средним поведением одного виброизолятора. Установлено, что для группы изоляторов наблюдается небольшое увеличение рабочей нагрузки. Также отклонения параметров приводят к снижению жесткости.
Выводы. Установленная высокая чувствительность виброизоляторов с квазинулевой жесткостью доказывает, что они требуют тщательного внимания и высокой точности при изготовлении. Полученные отклонения поведения группы виброизоляторов от поведения одиночного виброизолятора указывают на необходимость ввода соответствующих поправок при их проектировании, в противном случае это может привести к потере устойчивости и нестабильности положения оборудования, чего следует избегать.

Ключевые слова: вибрация, виброизолятор, квазинулевая жесткость, математическое моделирование, нелинейные колебания, чувствительность.

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